Evaluation of current distribution along the lightning discharge channel by a hybrid electromagnetic model

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Abstract

In this work, the influence of some physical characteristics of lightning channel on spatial and temporal return-stroke current distribution is considered. The results are obtained by simulation, employing a hybrid electromagnetic model. The effect of corona sheath and losses at the channel core on return-stroke current velocity and on current attenuation along the channel is evaluated.

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consider physical aspects, which are inherent to lightning, being only focused on the agreement between the generated electromagnetic fields predicted by the model and those observed by measurements at discharge vicinities.

In this paper, the authors described their efforts for understanding how two specific parameters (corona sheath and core losses) influence return-stroke current distribution and amplitude along discharge channel. For this task, they employed a hybrid electromagnetic model.

2. Some basic aspects

2.1. General comments

Lightning channel comprises two regions: an ionized core that determines a plasma path where return-stroke current flows and an external corona sheath. The characteristics of such regions may influence the temporal and spatial distribution of the lightning current.

A detailed approach of all aspects that influence on current distribution along channel is very complex, as it should also contemplate tortuosities and branches of lightning channel, height of attachment point above soil level, non-uniform propagation velocity of current wave along the channel, current reflections at soil level and influence of resistivity and orography. One particular aspect of interest concerns the understanding of the way corona sheath, losses in plasma core and channel tortuosities influence on return-stroke current propagation along channel. This paper refers to an investigation regarding this aspect.

2.2. Comments about developed simulation

The evaluations of this work were done by simulation with the application of an elaborated hybrid electromagnetic model. Though the presentation of such model does not constitute the objective of this paper, the model fundamental aspects are shown in Appendix A, in order to provide sufficient information for understanding its application. Details of this model are considered in another publications [5,6].

Corona effect and channel core losses present complex non-linear behaviour. Both aspects and also channel tortuosities influence current velocity and amplitude profile along discharge channel.

These factors were represented in the model assuming certain simplifications, considered to be consistent according to the nature of present evaluation.

Channel core losses were represented by attributing defined values to the resistance of unit length of channel core. The variation of losses was taken into account assuming different values for this resistance.

In order to compute corona effect, its sheath was replaced by an equivalent increase on channel radius. This representation corresponds to enlarge the equipotential surface, which is established around channel, just before the attachment and flow of return-stroke current. This kind of approach, computed
from “$qV$” curves, is verified to be consistent in experimental tests involving high-voltage electrodes. In the present case, as corona sheath is established in a relative slow process before attachment, the consistency of such approach seems to be improved. It is worthwhile to denote that this radius amplification is applied only for transversal current. This means that, for the longitudinal current that flows along the core, the radius remains the same of original core. So, to consider larger corona effect, the equivalent sheath radius should be increased.

Systematic simulations were implemented with such model. First, lightning channel was represented as a vertical current path and sensitivity analyses were performed in order to relate losses intensity and corona sheath extension to current wave velocity and attenuation. Following, the channel tortuosity was considered.

In simulations, a current wave (1/50μs ramp, 1 kA peak value) was supposed to be injected into channel at soil level. The current wave was observed 300 m above soil. The ramp curve was chosen only because it has very defined steepness and front time. This makes simple the analysis of current distribution along the channel.

3. Results and analyses

3.1. Channel core losses

In order to evaluate the influence of core losses, they were represented assuming different values for plasma conductivity. According to literature, after the attachment process, the core behaviour can be represented by a dynamic resistance, whose value depends on core ionization level. In his work [7], Rakov estimated this resistance to decrease from 3.5 Ω/m (ahead of the return-stroke front) to 0.035 Ω/m, after the channel has become well ionized (region behind the return-stroke front). For the subsequent return stroke, the channel has a higher ionization level and losses at channel core are smaller. In this work, lightning channel was simulated considering three different constant values for resistance: 0.035, 0.56 and 1 Ω/m. These values were computed by assuming different conductivities for the channel plasma and a constant value for core radius in all simulations. According to literature suggestion, such radius was assumed as 1 cm [8].

Fig. 1 shows the current observed at channel, 300 m above soil level for mentioned resistance values and denotes the influence of losses at channel core on lightning current. The increase of channel losses implies on a decrease of lightning current amplitude. For the wave corresponding to 0.035 Ω/m, the current profile is almost the same as the case in which conductivity value is so high as that of copper (∼$10^7$ S/m). In cases corresponding to 0.56 and 1 Ω/m, the amplitudes are, respectively, 10% and 15% lower than the previous case. Such reductions are more pronounced in the first 3 μs. These results also show that losses at the channel core, when considered alone, are not able to affect the current propagation velocity.
3.2. Corona sheath

During early stages of downward leader development, the high values of electric field around leader path establish intense corona activity. At time the attachment is achieved, such path is involved by a corona sheath.

In his approach [4,9], Cooray represents corona sheath, maintaining a commitment with the physical meaning of the lightning phenomenon. Leader channel is divided in two coaxial sections, one with a high conductivity ("hot corona sheath") and the other with a low one ("cold corona sheath"). The DU engineering model computes the contribution of corona sheath to the distribution of return-stroke current along channel, assuming current wave as the sum of two components. The first is associated to a fast discharge of the corona region and the other one to a slower discharge of corona sheath surrounding the channel core [10]. Another interesting approach is presented by Moini et al. Corona effect is computed by their electromagnetic model, assuming an artificial increase for the electric permittivity of surrounding medium [11].

In the present work, the influence of corona sheath was taken into account by an equivalent increase of channel radius (only for transversal current). Fig. 2 shows the obtained results, assuming three different values of equivalent radius.

The results show that corona sheath influences in two aspects. First, it promotes a reduction on the average velocity of return-stroke current to approximately 75%, 65% and 50% of light velocity, respectively, for equivalent corona radius of 2, 4 and 8 m. The velocity decrease may be perceived from the wave delay, in reference to the wave that propagates with no corona sheath, both observed at 300 m height. Second, it diminishes front wave steepness. The current wave amplitude is little affected. This result is in entire agreement with a physical analysis: the main corona effect corresponds to increase channel capacitance. If channel is approximated by a
transmission line, a capacitance increase would decrease surge propagation velocity \((v = 1/(LC)^{1/2})\). In the present case, inductance is considered constant, as the core section crossed by longitudinal current holds original radius.

3.3. Simultaneous representation: losses and corona sheath

Fig. 3 illustrates the results obtained when the effect of losses and a 2 m corona sheath are considered simultaneously.

The result confirms that losses are mainly responsible by attenuation of current wave amplitude and reduction of current velocity is mainly associated to corona
effect. Attenuation of current amplitude becomes more significant for the first 3 μs, when both aspects are simultaneously represented. Table 1 shows correspondent attenuation and current decay constant for each considered channel assumption.

Lightning current intensity decreases while propagating upward the channel. Fig. 4 shows the attenuation on current crest value along the channel, considering a 2 m corona sheath and $R = 0.035 \Omega/m$. For this condition, the estimated value for current decay constant is 3181 m, as shown in Table 1.

The traditional transmission line (TL) engineering model assumes the current wave to propagate along channel with no attenuation and distortion. The improved modified transmission line (MTL) model considers current amplitude to decay with height, though waveshape is supposed not to be affected [10]. A decay constant is defined to take into account the effect of charge deposited at the corona sheath [12].

### Table 1

Attenuation of current amplitude considering losses at channel core and a 2 m corona sheath (current waves observed 300 m above soil surface)

<table>
<thead>
<tr>
<th>Channel characteristics</th>
<th>Current amplitude (kA)</th>
<th>Attenuation (%)</th>
<th>Relative attenuation$^a$ (%)</th>
<th>Current decay constant (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No losses and no corona sheath</td>
<td>0.95</td>
<td>5</td>
<td>—</td>
<td>5848</td>
</tr>
<tr>
<td>2 m corona sheath, without losses</td>
<td>0.92</td>
<td>8</td>
<td>3.2</td>
<td>3598</td>
</tr>
<tr>
<td>2 m corona sheath, $R = 0.035 \Omega/m$</td>
<td>0.91</td>
<td>9</td>
<td>4.2</td>
<td>3181</td>
</tr>
<tr>
<td>2 m corona sheath, $R = 0.56 \Omega/m$</td>
<td>0.78</td>
<td>22</td>
<td>18</td>
<td>1208</td>
</tr>
<tr>
<td>2 m corona sheath, $R = 1 \Omega/m$</td>
<td>0.73</td>
<td>27</td>
<td>23</td>
<td>953</td>
</tr>
</tbody>
</table>

$^a$ Attenuation associated only to effect of channel core losses and corona sheath (in relation to the wave propagated in the condition of channel without losses and corona sheath).

![Fig. 4. Behaviour of current amplitude along channel (2 m corona sheath, $R = 0.035 \Omega/m$).](image)
According to Table 1, corona effect and core losses make this constant to range from around 1000 to 3600 m. This result seems consistent with the 2000 m adopted by Nucci et al. [13].

3.4. Three-dimensional representation of lightning channel

Usually, lightning channel is represented as a vertical conductor without branches and tortuosities. This condition is very different from the real phenomenon. Channel tortuosities are able to significantly influence current propagation, as commented by Rakov and Uman [3] and Le Vine and Willet [14].

The computed return-stroke current velocity is frequently obtained from two-dimensional or even one-dimensional (vertical) observation. The resultant apparent velocity determined in these cases is different from actual one. A 2000 m three-dimensional representation of a lightning channel was simulated in order to verify the influence of tortuosities on the apparent delay of current propagation. Light velocity \(c\) was assumed in order to avoid the influence of other effects. The channel geometry was estimated from a real photograph and is presented in Fig. 5. Also the current waves determined at two different heights are remarked.

The result denotes that a three-dimensional representation of lightning channel causes an apparent delay on current propagation. This would correspond to an average current velocity around 0.66 \(c\). This result is consistent with the conclusions of Idone and Orville [15], concerning the underestimation of velocity calculated from single photographs and the need for a three-dimensional return-stroke velocity determination.

4. Conclusions

This work addressed answers to some questions concerning the comprehension of how return-stroke current distribution and amplitude is influenced by corona,
channel core losses and tortuosity. In order to clarify such influence, simplifications were assumed for assuring a better evaluation of the nature of these aspects.

The employed model computed the variability of return-stroke current velocity and attenuation simply by assuming determined values for the resistance of unit length of channel core and an increase on the equivalent radius for corona sheath.

The evaluations showed that the reduction of current velocity is mainly associated to extension of corona sheath around the core, while attenuation of current wave amplitude along channel should be mainly attributed to losses in channel core.

The current decay constant was evaluated for specific conditions of corona and losses, as illustrated on Section 3.3.

Additionally, it was denoted that return-stroke current velocity estimated from a two-dimensional observation may be lower than the actual one, which should be associated to a three-dimensional representation of lightning channel.

Appendix A. Fundamental aspects of the hybrid electromagnetic model—HEM

HEM is an elaborated model for general application in lightning current associated problems. The present application allows some simplifications. This dedicated application is here commented. Model development comprises several steps:

(i) Current path (lightning channel) is partitioned in a large number of elements.
(ii) The electromagnetic couplings between every pair of elements is then calculated (mutual and self-couplings), employing expressions indicated in Fig. 6, derived from electric scalar and magnetic vector potentials. Calculations include skin effect and the soil effect (assumed as a perfect conductor in this specific application) by means of the traditional method of images. Also propagation effects are included.

(iii) Two linear systems are composed to relate (a) average potential of each element to its transversal current: $V = Z^T_T I_T$ (or $I_T = Y^T_T V$) and (b) voltage drop along each element to longitudinal current: $\Delta V = Z^L_L I_L$ (or $I_L = Y^L_L \Delta V$).

For any two elements ($i$ : victim and $j$ : field emitter) of length $L$:

- $V_{ij}$: average potential of element $i$ due to transversal current in $j$
- $Z_{ij}$: voltage drop along $i$ due to longitudinal current along $j$
- $\Delta V_{ij} = Z_{ij} I_{ij}$
- $Z_{ij} = \frac{1}{4\pi \mu \varepsilon} \int L \frac{e^{-kr}}{r} dl_i dl_j$

- $\sigma, \varepsilon, \mu$: air conductivity, electric and magnetic permeability; $r$: distance between $i$ and $j$; $k$: plane wave propagation constant; $\omega$: angular frequency.

Fig. 6. Representation of fundamental relations on HEM Model.
(iv) The electric potential of each node (point that connects two elements) is expressed as function of average potential and voltage drop for corresponding element: \( \Delta V_i = V_{Nk} - V_{Nm} \) and \( V_i = (V_{Nk} + V_{Nm})/2 \).

(v) Also transversal currents are supposed to derive from the nodes at the element extremities \( (I_{TNk} = I_{Ti}/2 \) and \( I_{TNm} = I_{Ti}/2 \)).

(vi) By manipulation of previous equations, new expressions are derived for \( I_T \) \( (I_T = Y_{TM} V_N) \) and for \( I_L \) \( (I_L = Y_{LM} V_N) \).

(vii) The two independent coupling systems are integrated. Current continuity principle is applied to each node \( (\sum I_N = 0; \) including external injected, transversal and longitudinal currents) and the longitudinal current that enters any specific node is expressed as the summation of all transversal currents after that node: \( I_{Li} = \sum I_T \) after.

(viii) As any current \( (I_{Li} \) and \( I_{Ti} \)) is expressed by the product of a line (from \( Y_{TM} \) or \( Y_{LM} \)) by the vector of potential at nodes, an only system of equations is composed when (vii) is applied: \( \Delta V_N = b \). \( b \) is the vector of external current injected at each node (only the first element, where current is injected, has no null value).

(ix) The system is solved by direct or indirect method and the node potentials \( (V_{Ni}) \) are all determined. From \( I_L \): \( I_L = Y_{LM} V_N \), longitudinal currents are immediately determined for all elements. Also all values for \( \Delta V_i \), \( V_i \) and \( I_T \) may be found.

The present application concentrates on evaluating the longitudinal current distribution along lightning channel for an external current, applied at soil level.

The described procedure is able to provide solution for a specific frequency. When applied to lightning currents, the first step is to find the frequency components of injected current by means of Fourier Transform. For each current component (considering its amplitude and phase), all longitudinal currents are found by the model. Applying Inverse Fourier Transform, all currents are found in time domain. Practical definitions, such as the element length and the range and number of frequencies to be employed, are done by the user, after some preliminary accuracy tests employing the same model.

The designation “HEM” is motivated by the adopted double approach: first, coupling relations are accurately evaluated from numerically implemented field equations; following the results are expressed by circuitual quantities (voltages and currents), which are related by the current continuity principle to provide the final answers.

References


