**AmpABL - Methodology of Ampacity Calculation for Overhead Line Considering the Effect of Atmospheric Boundary Layer**

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<th>Journal:</th>
<th><em>IEEE Transactions on Power Systems</em></th>
</tr>
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<td>Manuscript ID:</td>
<td>draft</td>
</tr>
<tr>
<td>Manuscript Type:</td>
<td>Transactions</td>
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<td>Date Submitted by the Author:</td>
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<td>DO NASCIMENTO, CARLOS; 1) UFMG - FEDERAL UNIVERSITY OF MINAS GERAIS 2) CEMIG, 1) ELECTRICAL ENGINEERING 2) OVERHEAD LINE MANAGEMENT</td>
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<td>Key Words:</td>
<td>Transmission lines, Terrestrial atmosphere, Power transmission meteorological factors, Power transmission reliability, Power system availability, Terrain mapping, Thermal factors, Wind</td>
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Abstract—This paper presents a new methodology for calculating the steady-state thermal rating of a given overhead line, which uses the wind speed data obtained from numerical analysis of the atmospheric boundary layer - ABL. The motivation behind this work is based on the difficulty and the high cost for monitoring the main weather parameters along the overhead line, especially the variation of wind speed. In the new methodology proposed in this paper, the spans with lower wind speed and lower clearance are defined as critical spans and, naturally, the thermal capacity of the overhead line is limited by them. The results obtained in applying this methodology in a 138 kV overhead line, with 133 spans, demonstrate the consistency of the proposed methodology and its applicability. Therefore it will be discussed how this new methodology can be applied both at design and at operation of overhead lines. In this case, the operation team will improve security with possible maximization of electric energy transmission.

Index Terms — Overhead Line, Weather Parameter, Steady-State Thermal Rating, Ampacity, Atmospheric Boundary Layer.

I. INTRODUCTION

An important point in improving the process of calculating the steady-state rating is the need to establish the main weather parameters such as wind speed, temperature and solar radiation [1] throughout all spans of the overhead line. The monitoring of weather conditions throughout the overhead line by weather remote stations has a high cost [2], especially for long lines. Thus, a plausible starting point to work in determining the wind speed, which is the parameter of major weather influence in the calculation of thermal rating, is the numerical study of atmospheric boundary layer - ABL [3], [4] around the overhead line.

In order to validate the model of the ABL, it is necessary to have the digital maps of topography and measurements of wind speed at some points on the boundary of the region and inside the ABL. Once the model is validated, the next step is to generate a database by simulations with different boundary conditions and then to apply it in the determination of the worst weather spans, i.e. spans with the lowest values of wind speed. It is important to mention that there is still no methodology proposed in the literature for finding critical spans in Overhead Lines [5]–[7].

In this context, the main goal of this work is to present a new methodology for calculating the steady-state rating - AmpABL, which is more realistic than the traditional methods of calculation [7], [9]. In the presentation that follows, are briefly presented the method for calculating the steady-state thermal rating, the proposed criterion for determining the critical span, an introduction of the ABL model, a detailed description of the methodology AmpABL and finally its application that was originally proposed in this work.

II. STEADY-STATE THERMAL RATING CALCULATION

The IEEE Std 738™-2006 defines the term “steady-state thermal rating” as "the constant electrical current that would yield the maximum allowable conductor temperature for specified weather conditions and conductor characteristics under the assumption that the conductor is in thermal equilibrium (steady state)". Considering only the most significant terms in the thermal equilibrium as in (1),

\[ P_j + P_s = P_s + P_c \]  

(1)

it is possible to determine what is the maximum current that can flow in a conductor when considering the rate of loss by Joule effect. That is,

\[ R(T_c)I^2 + P_s = P_s + P_c \]  

(2)

Then,

\[ I = \sqrt{\frac{P_s + P_c - P_s}{R(T_c)}} \]  

(3)

Through (1)-(3), \( P_j, P_s, P_R \) and \( P_C \) are respectively the rate of heat gain by Joule effect, the rate of heat gain by solar radiation, the rate of heat loss by radiation and the rate of convective heat loss (W/m). The parameters \( R \) and \( T_C \) are the resistance per unit length (Ω/m) and the conductor temperature (°C).

The steady-state thermal rating calculation of overhead line is usually done for a maximum allowable conductor temperature specified in the design stage \( T_C < 100°C \) and conservative weather conditions, that is, bounds of both wind speed of [0.6,1.2] (m/s) and summer temperature [30,45] (°C).
III. Steady-state Conductor Temperature Calculation

The rates of heat loss by convection and radiation are not linearly dependent on the temperature of the conductor, thus the equation of thermal balance (1) can be solved to determine the temperature of the conductor in terms of the current and the weather variables through an iterative process [10], i.e. according to the following steps:

a) Take an initial conductor temperature $T_R$, the current $I_C$ for which the conductor temperature is expected and a small tolerance number $\varepsilon$.

b) Calculate the corresponding heat losses $P_R$ and $P_C$, the heat gain by solar radiation $P_S$ and the conductor resistance $R(T_R)$.

c) Calculate the conductor current ($I_K$) for the reference temperature ($T_K$) using (3).

d) Compare the values of $I_C$ and $I_K$.
   - If $|I_C-I_K| \leq \varepsilon$, stop the iteration and consider the $T_R$ value as the solution, i.e., $T_C = T_R$.
   - If $|I_C-I_K| > \varepsilon$, increase $T_R$, otherwise $T_R$ is decreased.

e) Return to step “b”.

IV. Criteria for Determining Critical Spans

The determination of critical spans is an important task because if an electrical failure occurs, the major probability is that it will take place in one of these spans. The definitions of critical spans, with the factors that influence them, are in a refined manner discussed in [2]. The critical spans are basically those with lowest values of wind speed and concurrently lowest clearance between conductor and nearest grounded object.

In the proposed AmpABL methodology, the determination of critical spans is carried out in an original and low cost way if compared to the cost of instrumenting every span, which is necessary for monitoring the whole transmission line, as mentioned in [2]. Basically, the difference is that the values of wind speed considered in the proposed methodology is obtained from simulations of atmospheric boundary layer, with prescribed boundary conditions (wind speed = 1 m/s at different directions of incidence). The spans that have numerical results of wind speed less than 1 m/s are considered worst weather spans. The reference value of 1m/s for the wind speed at the boundary surfaces is a technical criterion based on the region of the world considered. Of course, this value should be different for other region in the world.

Mathematically, let $S_{we}$ be the set of critical spans with respect to electrical clearance [11], i.e., the set composed by spans with clearance less than a certain clearance value of reference, which is dependent of the transmission line design. Let $S_{ww}$ be the set of worst weather spans, i.e., the set composed by spans with numerical results of wind speed normal to the axis of the conductor lower than the reference value of 1 m/s.

Note that a span belonging to $S_{ww}$ has a higher probability of violating the minimum safety clearance compared to another not belonging to $S_{ww}$. Therefore, one span of $S_{ww}$ will have a greater probability of electrical failure. Furthermore, the spans that belong to $S_{ww}$ have less cooling capacity of the conductor and therefore greater thermal risk.

Now, let $S_{ww}^{\text{c}}$ be the set of spans obtained by the intersection of $S_{ww}$ and $S_{we}$ as in (4).

$$S_{ww}^{\text{c}} = S_{ww} \cap S_{we} \quad (4)$$

Eliminating from the $S_{ww}^{\text{c}}$ the dominated spans by applying a non-dominance criterion, the result is a set of non-dominated spans with small clearance and wind speed. Let this set of non-dominated spans be $S_c$.

Note that the spans belonging to $S_c$ form a non-dominated frontier in the space “wind speed versus clearance”. They are those with simultaneously smallest clearances and lowest wind speeds, among all spans of the overhead line. That is, they are strong candidates to be the critical critical spans in terms of risk of failure and thermal risk. Once determined the set $S_c$, it is necessary to determine the critical spans, by calculating the conductor temperature of each span and their new corresponding clearance. The critical span is the one that would always be hottest and/or nearest to clearance limits for a given current loading.

V. Atmospheric Boundary Layer - ABL

The region of ABL is defined as a thin layer of the atmosphere, adjacent to the surface (up to 2 km in height), where wind speed show a high Reynolds number. This flow of wind occurs at different scales, each scale is described in terms of computational domain with a different model. These models use the same governing equations [3], showing differences only in terms of simplifications of source, which depends mainly on the computational domain and the thermal stratification. The equations governing these flows are the geophysical equations of continuity, conservation of momentum, conservation of energy and conservation of chemical species. The wind speed simulation models are classified into four categories: global circulation, weather forecast, meso-scale and micro-scale. The global circulation model is used in domains between 200 and 500 km along the earth’s surface and is used to analyse the vector of wind speed. The models for weather forecast are used for domains between 50 and 100 km to solve problems of weather fronts. The meso-scale models are used typically for domains from 2 to 50 km and employed in the wind speed studies influenced by the topography, and finally the micro-scale models are used for small specific domains and employed in studies where the wind is influenced by the regionalization of the earth's surface.

The numerical study of the ABL in complex topography for use in climatological studies was introduced in [4]. The details of modelling the boundary layer are presented in Appendix A.

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1 One vector $\mathbf{u} = (u_1, \ldots, u_i)$ dominates another vector $\mathbf{v} = (v_1, \ldots, v_i)$ (represented by $\mathbf{u} \leq \mathbf{v}$) if and only if $\mathbf{u}$ is partially less than $\mathbf{v}$, i.e., $\forall i \in \{1, \ldots, k\}, u_i \leq v_i$ and $\exists i \in \{1, \ldots, k\} | u_i < v_i$. If the vector $\mathbf{u}$ is not-dominated by any other vector in the space, the vector $\mathbf{u}$ is a non-dominated vector.
A. ABL Simulation

The ABL simulation was performed under the use of ANSYS CFX computational system [12]. Only four directions of the wind were considered at angles of 45°, 135°, 225° and 315° (see Fig. 1). All these simulations were performed using on the inlet surface (see Fig. 6), a logarithmic boundary condition with wind speed equal to 1 m/s at 10 m height from the ground and neutral atmosphere, i.e. the Froude number exceeds 1000 (as in (11)). Analysing the numerical results, the set of spans belonging to \( S_{ww} \) was identified, i.e., spans with wind speed less than 1 m/s. Of course, many other angles of incidence could have been considered to precisely identify the spans with lowest wind speed.

VI. AMPABL METHODOLOGY

The methodology proposed in this paper was developed based on information of wind speed (obtained from ABL simulation) and span clearance of the whole transmission line (obtained from the transmission line design). AMPABL methodology can be summarized in the following set of steps:

1) Select the appropriate parameters of the overhead transmission line to be analysed.
2) Select the region of study (by digitalizing the topographic region containing all spans of the overhead line) and discretize it using an appropriate mesh generator.
3) Carry out simulations of the ABL as described in Section V.
4) Identify the spans to constitute the set \( S_{ww} \), as described in section IV.
5) Identify the spans to compose the set \( S_{we} \), as described in the Section IV.
6) Take the intersection between \( S_{ww} \) and \( S_{we} \) to have the set \( S_{wwe} \). Eliminate the dominated spans to have the non-dominated ones. Store the identification number of non-dominated spans in a vector \( S_c \).
7) Calculate the conductor temperature for all spans belonging to \( S_c \), using the iterative method described in section III, considering the corresponding wind speed determined in step 4.
8) Recalculate the clearances for all spans of \( S_c \).
9) Identify the critical spans, i.e., least clearance and/or hottest conductor as described in section III.
10) Calculate the steady-state thermal rating for the set of critical spans identified in step 9.

The advantage of AmpABL methodology is to allow the identification of critical spans without monitoring any spans of the whole overhead line. The monitoring and calculation of the dynamic rating in real time by conductor temperature sensors are now possible to be made simply monitoring the critical spans. The financial cost to supervise them is expected to be lower if compared to other methodologies.

VII. APPLICATION OF AMPABL

The proposed methodology AmpABL was applied in determining the steady-state thermal rating [10] of a real overhead line designed for the following parameters:

1) Conductor Linnet or 336 MCM
2) Line Voltage = 138 kV
3) Solar radiation = 1000 W/m²
4) Design wind speed of 1 m/s at 10 m height from the ground
5) Angle between the wind speed direction and the conductor axis equal to 90 degrees
6) 1 PU = 510 A
7) Ambient temperature = 30 °C
8) TN: Normal Design Temperature of Conductor = 70 °C
9) TE: Emergency Design Temperature = 90 °C

A. Step 1: Parameters of the Overhead Line

The overhead line has 133 spans distributed along 52 km of length in the region of Acuruí-MG, southeast of Brazil. The minimum clearances, in all spans, were extracted directly from the design or calculated by using PLS-CADD™ [11]. For purposes of illustration one span is shown in Fig. 2.
B. Step 2: Mesh Generation

The region of interest was properly discretized using an appropriate mesh generator [12]. The Fig. 3 shows the mesh of part of the region containing part of the overhead line.

Fig. 3. Discretized domain with part of the overhead line.

C. Step 3: ABL Simulation

The simulation was performed for each of the 4 wind directions (45°, 135°, 225° and 315°) with the wind speed on the inlet surface (see Fig. 6) equal to 1 (m/s) at 10 m height using a logarithmic profile (see (11)). The numerical wind speed on each span was then extracted from the numerical results. The lowest numerical wind speed result obtained among the four simulations was considered as the minimum wind speed at the corresponding span. Table I shows the results for a few spans.

<table>
<thead>
<tr>
<th>Span</th>
<th>Incidence Angle of Wind Speed at the Conductor (Fig. 1) (°)</th>
<th>Lowest Numerical Wind Speed Result (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19-20</td>
<td>45</td>
<td>3.8</td>
</tr>
<tr>
<td>19-20</td>
<td>135</td>
<td>1.4</td>
</tr>
<tr>
<td>19-20</td>
<td>225</td>
<td>1.1</td>
</tr>
<tr>
<td>19-20</td>
<td>315</td>
<td>4.8</td>
</tr>
<tr>
<td>20-21</td>
<td>45</td>
<td>3.5</td>
</tr>
<tr>
<td>20-21</td>
<td>135</td>
<td>1.2</td>
</tr>
<tr>
<td>20-21</td>
<td>225</td>
<td>0.9</td>
</tr>
<tr>
<td>20-21</td>
<td>315</td>
<td>4.1</td>
</tr>
<tr>
<td>21-22</td>
<td>45</td>
<td>3.7</td>
</tr>
<tr>
<td>21-22</td>
<td>135</td>
<td>1.2</td>
</tr>
<tr>
<td>21-22</td>
<td>225</td>
<td>1.0</td>
</tr>
<tr>
<td>21-22</td>
<td>315</td>
<td>3.8</td>
</tr>
</tbody>
</table>

D. Step 4: Span Identification to Constitute the Set $S_{we}$

The Fig. 4 shows the numerical results of wind speed at each span. These values are the lowest winds speeds obtained from the simulations as described early. It is observed that only 3 spans present wind speed lower than the reference value of 1 m/s. On these spans with lower wind speed it is expected the highest conductor temperatures, reduction of clearance and augmentation of both thermal and electrical failure risks.

E. Step 5: Span Identification to Constitute the Set $S_{we}$

The detection of which spans have small electrical clearance to constitute the set $S_{we}$ is obtained from the electromechanical design of the overhead line. This is made taking into account the clearance between conductor and ground (or grounded object if nearest to the conductor), and then checking whether this distance is less than a certain value of reference ($h_{ref}$), which should be higher than the minimum safety clearance specified in the project ($h_{min}$). For example, a clearance of $h_{ref} = 10$ (m) is the minimum safety distance for lines of 138 kV. Table III shows the results of this analysis based on safety standard requirements for design. It is observed that among the 133 spans, only 5 of them have heights equal to 10 (m). None of the spans of this overhead line can have a height less than this reference value.

<table>
<thead>
<tr>
<th>Span</th>
<th>Numerical results of Wind Speed (m/s)</th>
<th>Clearance (m) for Design Wind Speed equal 1 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-21</td>
<td>0.9</td>
<td>10</td>
</tr>
<tr>
<td>40-41</td>
<td>0.9</td>
<td>10</td>
</tr>
<tr>
<td>51-52</td>
<td>0.9</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Span</th>
<th>Simulated Wind Speed by ABL (m/s)</th>
<th>Clearance (m) for Design Wind Speed equal 1 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-4</td>
<td>1.1</td>
<td>10</td>
</tr>
<tr>
<td>19-20</td>
<td>1.1</td>
<td>10</td>
</tr>
<tr>
<td>20-21</td>
<td>0.9</td>
<td>10</td>
</tr>
<tr>
<td>39-40</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>40-41</td>
<td>0.9</td>
<td>10</td>
</tr>
</tbody>
</table>
F. Steps 6, 7, 8 and 9

The intersection between the sets $S_{ww}$ and $S_{we}$, i.e., $S_{ww} \cap S_{we}$, results in the following two spans ([20-21], 40-41). When applying the criterion of non-dominance on this set by considering the pairs of (wind speed, clearance), it is possible to realize that these spans have same values of wind speed and clearance. As the wind speed is less than the one considered during the overhead line design, new calculations of the conductor temperature and consequently for the clearance are necessary. After performing this calculus the results obtained are respectively 72.6 (°C) and 9.9 (m). Both these values violate the limits required by the electromechanical design of this overhead line [70 (°C) and 10 (m)].

Another alternative analysis to determine the non-dominated set is to plot the data from all spans $S_{ww}$ and $S_{we}$ in a graph of wind speed versus clearance, after correcting the temperature and clearance and taking the non-dominated points. Fig. 5 shows this procedure considering the number of spans belonging to $S_{ww}$ and $S_{we}$. Clearly the same final results are obtained, that is, clearance of 9.9 (m).

![Graph of wind speed versus clearance](image)

Fig. 5. Identification of non-dominated spans.

G. Step 10: Deterministic Thermal Rating in Critical Spans - Sc

The steady-state thermal rating for the critical spans defines the maximum capacity of the overhead line. In this manner, for the wind speed of 0.9 (m/s) and conductor temperature equal to 70 (°C) from the overhead line design, the maximum capacity results in 0.965 (PU). This result is 3.5% smaller than the original ampacity which was calculated for the design with wind speed equal to 1 (m/s) and conductor temperature equal to 70 (°C).

This result shows that the overhead line monitoring could be done in real time just on the two critical spans. This monitoring should be done via sensors for measuring the conductor temperature or clearance. This information is certainly of great value for the company, because the whole overhead line monitoring would be much expensive.

Of course, more ABL simulations with many wind direction in the boundary condition is worthwhile. For example, if more data from ABL simulations are considered, with small angles between adjacent main wind directions, results much more precise are expected. However, the few simulations presented in this paper are intended to show the application of the proposed AmpABL in a real overhead line.

This methodology can be applied both during the design and the operation of the overhead line. At the design, the proposed methodology allows the designer to get a realistic view of the line when the critical spans are found. At the operation, the AmpABL allows the monitoring of these critical spans. Moreover, applying this methodology, are expected that both the reliability and transmission capacity are improved.

VIII. Conclusion

The AmpABL methodology was presented in details in this work. The core of this methodology is the procedure to find the critical spans which define the overall capacity of electrical energy transmission. The methodology was applied in one real overhead line of 138 kV, with 133 spans in the region of Acuruí (southeast-Brazil). This application was discussed step by step to contrast the main characteristics of the proposed methodology. The analysis of the results first pointed out that it is possible, after applying the AmpABL, to supervise the overhead line in real time with small investment (2 critical spans found among 133 ones) and finally that the maximum capacity or thermal rating of this overhead line could be better explored. One expects that this methodology is global because the usual way of designing overhead transmission lines is by using the steady-state procedure. In this procedure, the allowable maximum conductor temperature, wind speed, weather conditions, for the whole overhead line, are defined in a conservative way.

APPENDIX A – MATHEMATICAL MODEL OF ABL

The mathematical model of the ABL is determined by equations (5) to (8). They represent, respectively, conservation of mass or continuity equation (5), momentum (6), state equation (7) and thermal energy (8) based on Reynolds averaging and Boussinesq approximation [15], [17], [18].

$$\frac{\partial \hat{u}_i}{\partial x_i} = 0$$  (5)

$$\frac{\partial \hat{u}_i}{\partial t} + u_j \frac{\partial \hat{u}_i}{\partial x_j} = - \frac{\partial}{\partial x_i} \left( \frac{\delta \rho}{\rho_0} + \frac{2k}{3} \right) + \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - S_w$$  (6)

$$\frac{\delta \rho}{\rho_0} = - \frac{\partial T}{T_0}$$  (7)

$$c_p \left( \frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left( \frac{\nu}{P_r} \frac{\partial T}{\partial x_j} \right) + Q$$  (8)

where $u_i$ is i-th mean wind speed component (m/s), $\delta \rho$ the deviation of fluid density from its reference value $\rho_0$, $\rho_0$ the fluid density (kg/m$^3$), $\delta \rho$ is the deviation of pressure density...
from its reference value \( p \) (N/m\(^2\)), \( k \) turbulent kinetic energy (TKE) (m\(^2\)/s\(^2\)), \( t \) the time (s), \( \nu \), the effective kinematic viscosity (m\(^2\)/s), \( S \), the source term (m/s\(^2\)), \( C_p \) the specific heat or air at constant pressure (dimensionless), \( T \) the temperature (°C), \( Pr \), the Prandt number of turbulence (dimensionless) and \( Q \) the heat generation rate.

The term \( S \) represents a flotation term, which considers turbulence effect. This fluctuation characterizes many models available in literature. Usually, the models presented in [13]-[15] use the Boussinesq approximation and the models of turbulence are derived from \( k-\varepsilon \) model.

### A. Domain of Analysis

The simulation of Atmospheric Boundary Layer - ABL is made on a domain that has its base on the ground. The vegetation, water, or buildings impose difficulties for the movement of the air layer. The top of the layer must be high enough, in order to be considered boundary conditions of Neumann natural type, without loss of accuracy in results. At the inlet, are imposed both the turbulent \( k-\varepsilon \) model and the logarithmic distribution of wind speed profile. On both sides, are also imposed the condition of symmetry and in the outlet the outlet condition. Fig. 6 shows all sides of the domain where the boundary conditions should be defined.

![Illustration of boundary conditions.](image)

### B. Boundary Conditions

1) **Boundary Condition: \( k-\varepsilon \) Model**

The boundary conditions used in the ABL simulation by the model of turbulence \( k-\varepsilon \) are associated with the turbulent kinetic energy \( k \) and its dissipation rate \( \varepsilon \). In this model, the inlet conditions are given by (9) and (10).

\[
k = \frac{u_*^2}{\sqrt{C_p}}
\]

(9)

\[
\varepsilon = \frac{u_*^3}{\kappa x_2}
\]

(10)

where \( u_* \) is the wind speed of friction (m/s), \( \kappa \) Von Karman constant (\( \kappa = 0.41 \)) (dimensionless), \( C_p \) empirical constant (dimensionless) and \( x_2 \) is the height (m).

2) **Boundary Condition at Inlet**

The logarithmic wind speed at inlet of ABL is given by (11) based on height \( x_2 \).

\[
u(x_2)_{in} = u_{ref}.\ln\left(\frac{x_2}{x_{2ref}}\right) / \ln\left(\frac{x_{2ref}}{x_{2o}}\right)
\]

(11)

where \( u(x_2)_{in} \) is the wind speed (m/s), \( u_{ref} \) wind speed of reference at 10 m in height (m/s), \( x_{2ref} \) reference height equal to 10 (m) and \( x_{2o} \) the aerodynamic roughness length (m).

Typical values of roughness are presented in Table IV.

<table>
<thead>
<tr>
<th>( Z_0 ) (m)</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>City</td>
</tr>
<tr>
<td>0.3</td>
<td>Forest</td>
</tr>
<tr>
<td>0.03</td>
<td>Low grass</td>
</tr>
<tr>
<td>0.0001</td>
<td>Water</td>
</tr>
</tbody>
</table>

### 3) Outlet and Lateral Boundary Condition

This boundary condition is given by natural Neumann condition, that is,

\[
\frac{\partial u_i}{\partial x_j} = 0 \quad \forall i = 1,2,3
\]

(12)

where \( j \) is the direction normal to the surface, that is, \( j = 1 \) for the outlet surface and \( j = 3 \) for the lateral ones.

### 4) Wall Boundary Condition

The wind speed in the ground is considered without friction, that is, all speed components are null, that is,

\[
u_i = u_j = 0
\]

(13)

### 5) Upper Boundary Condition

All the boundary conditions are Neumann type except for the main speed component that is Dirichlet boundary condition,

\[
u = U_w
\]

(14)

Table V shows the boundary conditions that should be imposed on the domain boundaries.
C. Treatment of Vertical Buoyancy Strength

The strength of vertical buoyancy is considered as a source term $S_w$ in the momentum equation (5), which is given by

$$ S_w = \frac{g \cdot \left( \rho - \rho_0 \right) \delta_{x_2}}{\rho_0} \delta_{2i} $$

(15)

where $S_w$ is the source term by unit of fluid density given in (m/s²), $g$ gravity acceleration (m/s²), $\rho$ fluid density (kg/m³), $\rho_0$ the fluid reference density (kg/m³) and $\delta_{2i}$ is the Kronecker delta which is unity only for $i$ equal to 2 and zero for $i = \{1,3\}$. When considering the density variation $\Delta \rho$ constant during the simulation, the Froude number can be calculated by (16).

$$ Fr = \frac{U_x}{\left( g \cdot L \cdot \Delta \rho / \rho_0 \right)^{1/2}} $$

(16)

where $U_x$ is the wind speed above 500 meters of height (m/s), $\Delta \rho$ variation between the base of the land to its top (kg/m³) and $L$ the height of the simulation domain (m).

In this way, the source term on the $x_2$ direction (height), introduced in the equation of momentum can be written as (17).

$$ S_w = \frac{U_x^2}{L \cdot Fr^2} \delta_{2i} $$

(17)

Finally, the intensity of the strength of buoyancy due to thermal effects of the ground is obtained from values assigned to the Froude number. Table VI shows the relationship between the state of the atmosphere and the range of values assigned to the Froude number.

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ACKNOWLEDGMENT

The authors thank Dr. Ramon Molina Valle by the technical support in the ABL analysis and the electrical engineers Saulo Mariano and Marcello Dantas, and the undergraduate students Moisés Ferber and Alexandre Sifuente by the ABL simulations.

---

REFERENCES


BIOGRAPHY

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